Math 1411 Quiz 4 Name KEY

You may use a calculator and your homework, but not your books or notes. There are three (3) problems worth 10 points each. **Show all of your work to receive full/partial credit.**

1) (#25 from 3.1) Locate the absolute extrema of the function on the closed interval.

$$g(t) = \frac{t^2}{t^2+3}, \quad [-1,1]$$

$$g'(t) = \frac{2t(t^2+3) - t^2(2t)}{(t^2+3)^2} = \frac{2t^3+6t-2t^3}{(t^2+3)^2} = \frac{6t}{(t^2+3)^2}$$

$6t = 0 \rightarrow t = 0$

$g(-1) = \frac{(-1)^2}{(-1)^2+3} = \frac{1}{4}$

$g(0) = 0$

$g(1) = \frac{1}{4}$

abs. max. is $\frac{1}{4}$

abs min. is 0

2) (#39 from 3.2) Determine whether the Mean Value Theorem can be applied to f on the closed interval $[a, b]$. If the Mean Value Theorem can be applied, find all values of c in the open interval (a, b) such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.

$$f(x) = x^3 + 2x, \quad [-1,1]$$

$f(1) = 3$

$f(-1) = -3$

M.V.T. can be applied - $f(x)$ is continuous & differentiable on given interval.

$$\frac{f(1)-f(-1)}{1-(-1)} = \frac{3-(-3)}{2} = \frac{6}{2} = 3$$

$f'(x) = 3x^2+2$, so $3x^2+2 = 3 \rightarrow 3x^2 = 1$

$x^2 = \frac{1}{3} \rightarrow x = \pm\sqrt{\frac{1}{3}} = \pm\frac{\sqrt{3}}{3}$ $(\approx \pm 0.578)$

3) (#25 from 3.3) Consider the function

$$f(x) = \frac{x^5 - 5x}{5} = \frac{1}{5}x^5 - x$$

a) Find the critical numbers of f (if any).

b) Find the open interval(s) on which the function is increasing or decreasing.

c) Apply the First Derivative Test to identify all relative extrema.

$f'(x) = x^4 - 1 \rightarrow x^4 - 1 = 0 \rightarrow x^4 = 1 \rightarrow x = \pm 1$

a) critical numbers are $x = 1$, $x = -1$

b)

	$(-\infty, -1)$	$(-1, 1)$	$(1, \infty)$
T.V.	$x = -2$	$x = 0$	$x = 2$
sign of f'	$+$	$-$	$+$
Conclusion	$\uparrow$	$\downarrow$	$\uparrow$

increasing on $(-\infty, -1) \cup (1, \infty)$

decreasing on $(-1, 1)$

c) There's a max. at $x = -1$ and a min. at $x = 1$

$f(-1) = \frac{(-1)^5 - 5(-1)}{5} = \frac{-1+5}{5} = \frac{4}{5}$, max is at $(-1, \frac{4}{5})$

$f(1) = \frac{(1)^5 - 5(1)}{5} = -\frac{4}{5}$, min. is at $(1, -\frac{4}{5})$